Published by Institute of Physics Publishing for SISSA/ISAS
Received: *September 4, 2003*
Revised: *December 16, 2003*
Accepted: *December 17, 2003*

Large transverse momenta in statistical models of high energy interactions

Tadeusz Wibig
Experimental Physics Dept., University of Łódź
Pomorska 149/153, 90–236 Łódź, Poland, and
The Andrzej Sołtan Institute For Nuclear Studies, Cosmic Ray Lab., Łódź,
Uniwersytecka 5, POB 447, Łódź 1, Poland
E-mail: wibig@zpk.u.lod.edu.pl

Izabela Kurp
The Andrzej Sołtan Institute For Nuclear Studies, Cosmic Ray Lab.
Łódź, Uniwersytecka 5, POB 447, Łódź 1, Poland
E-mail: kurp@zpk.u.lodz.pl

Abstract: The creation of particles with large transverse momenta in high energy hadronic collisions is a long standing problem. The transition from small- (soft) to hard-parton scattering 'high-$p_\perp$' events is rather smooth. In this paper we apply the non-extensive statistical framework to calculate transverse momentum distributions of long lived hadrons created at energies from low ($\sqrt{s} \approx 10\,\mathrm{GeV}$) to the highest energies available in collider experiments ($\sqrt{s} \approx 2000\,\mathrm{GeV}$). Satisfactory agreement with the experimental data is achieved. The systematic increase of the non-extensivity parameter with energy found can be understood as phenomenological evidence for the increased role of long range correlations in the hadronization process.
Predictions concerning the rise of average transverse momenta up to the highest cosmic ray energies are also given and discussed.

Keywords: Jets, Phenomenological Models.

 http://jhep.sissa.it/archive/papers/jhep122003039/jhep122003039.pdf

Contents

1. Introduction

In what follows we write the momentum, p, in units of energy (e.g. GeV); strictly, of course it should be in units of energy divided by c.

For soft (low $p_\perp$ events) particle production, the QCD-based string model predicts that the transverse momentum (transverse mass: $m_\perp = \sqrt{{p_\perp}^2 + m^2}$) distribution of the produced quarks should be, in general, of the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}{p_\perp}^2} \sim \mathrm{e}^{-\pi {m_\perp}^2/\kappa^2} \,, \tag{1.1}$$

where κ is the string tension. According to [1], when the string tension may fluctuate, the string hadronization becomes consistent with thermal behaviour, and

$$\frac{\mathrm{d}\sigma}{\mathrm{d}{p_\perp}^2} \sim \mathrm{e}^{-\pi m_\perp/T} \,. \tag{1.2}$$

This means that in the soft (small $p_\perp$) region, the partonic string fragmentation picture, which is based on first principles, can be, to some extent, successfully replaced by the "phenomenological" statistical model.

Much more interesting, and complex, is the case of high transverse momentum physics.

The main reason for the failure of the traditional thermodynamical models of multiparticle production [2, 3, 4] was the experimentally observed significant increase of the production of particles with high transverse momenta discovered in the mid-seventies in the Intersecting Storage Ring (ISR) experiments. Its interpretation in the framework of jet models [5, 6] strongly supported the parton idea. But then new problems arose. The "hard" parton scattering expected from field theories implies roughly that the momenta should fall off as ${p_\perp}^{-4}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}{p_\perp}^2} \sim F_A(x_a,\, q_\perp^2)\, F_B(x_b,\, q_\perp^2)\, \frac{\alpha_s^2(q_\perp^2)}{q_\perp^4} \tag{1.3}$$

(where the F's are the respective structure functions). However the power-law fit to the data is closer to ${p_\perp}^{-8}$, a fact that was realized already in, e.g., [7] and [8].

The commonly accepted explanation of high $p_\perp$ behaviour in inclusive pp (and $p\bar{p}$) spectra is that at high enough energies the QCD hard scattering effects start to play a significant role. The transition "soft"→"not-so-hard"→"hard" *is believed* to combine the exponential low-$p_\perp$ domain with the asymptotic ${p_\perp}^{-4}$ tail in such a way that the expected distributions are, around a few GeV/c and for the interaction energies available at present, just as observed. To follow this idea in detail further QCD calculations are needed, and to cover the whole $p_\perp$ range some interplay of perturbative (high $p_\perp$-"hard") and non-perturbative ("soft") physics have to be developed in a self-consistent way. The problem is not trivial and here we point a way forward.

In the present paper we will show that the problem can be quite satisfactorily solved by way of statistical language as an effect of multi-particle, long-range correlations. We *believe* that both descriptions are equivalent.

In the standard statistical (thermodynamical) models of hadronization, high $p_\perp$'s can be explained (?) only by introducing additional mechanisms. Two such attempts have appeared recently.

One way [9] is to smear the $p_\perp$s with an additional, dynamic, term describing the collective motion of different parts of a "pre-hadronizing" state of matter. It can be achieved by introducing a distribution in momentum space of many fireballs created in the collision. However this mechanism cannot be responsible for very high $p_\perp$ tails.

The second idea [10, 11, 12] is to allow the Hagedorn temperature to fluctuate around its mean. A very good description of the data can be obtained assuming that [11, 12]

$$f(\beta) = p(1/T) = \frac{\alpha^\lambda}{\Gamma(\lambda)} \left(\frac{1}{T}\right)^{\lambda-1} \exp\left(-\frac{\alpha}{T}\right) \tag{1.4}$$

with parameters

$$\left\langle \frac{1}{T} \right\rangle = \frac{\lambda}{\alpha}, \qquad \left\langle \frac{1}{T^2} \right\rangle - \left\langle \frac{1}{T} \right\rangle^2 = \frac{\lambda}{\alpha^2}\,. \tag{1.5}$$

It is clear that the relatively narrow Γ distribution in $\beta = 1/T$ gives necessary very substantial tails in the distribution of T. The situation is presented in figure 1. In fact, interesting cases with high $p_\perp$ originate in events with actual Hagedorn temperatures much greater than the "hadronic soup boiling temperature" [13]. This is rather inconsistent with the general Hagedorn fireball picture.

In the present work we modify the classical Hagedorn idea by introducing the long-range correlations in statistical way.

JHEP12(2003)039

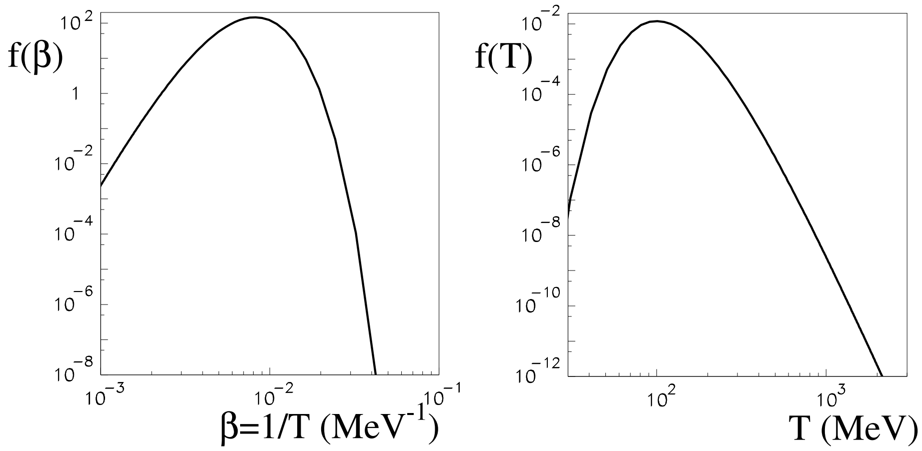

Figure 1: Distribution of $\beta = 1/T$ (left) and T (right) in the picture of $p_\perp$ - broadening by temperature fluctuations with $\lambda = 10$ and $T_0 = \langle 1/T \rangle^{-1} = \alpha/\lambda = 110\,\mathrm{MeV}$.

2. A standard statistical model

One way to describe the statistical properties of the system is by introducing the concept of the partition function. Its classical definition for the system in the state described by the vector $\mathbf{Q}_0$ (*canonical* statistical ensemble) is given by

$$\mathcal{Z}(\mathbf{Q}) = \sum_{\text{states}} \mathrm{e}^{-E/T} \delta_{\mathbf{Q},\mathbf{Q}_0} \,, \tag{2.1}$$

where E is the energy of the system. For the purposes of the present paper, where the system contains hadrons created in high energy hadronic collisions, Q can be limited to a three dimensional vector $\mathbf{Q}$ and its components are the total charge, the baryon number and the strangeness of the system (Q, B, S). Of course, when needed, a generalization is straightforward.

We follow here the formalism developed by Becattini (see, e.g., [14]):

By using the integral representation of the Kronecker δ factor, eq. (2.1) becomes

$$\mathcal{Z}(\mathbf{Q}) = \sum_{\text{states}} \frac{1}{(2\pi)^3} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \mathrm{d}^3\phi \, \mathrm{e}^{-E/T} \, \mathrm{e}^{i(\mathbf{Q}_0 - \mathbf{Q})\,\phi} \,. \tag{2.2}$$

If we have a quantum gas with N_b boson and N_f fermion species a summation over states can be performed, giving

$$\mathcal{Z}(\mathbf{Q}) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3\phi \mathrm{e}^{i\mathbf{Q}_0\phi} \exp\left[\sum_{N_b} \sum_k \log\left(1 - \mathrm{e}^{-\frac{E_k}{T} - i\mathbf{q}_k\phi}\right)^{-1} + \right.$$
$$\left. + \sum_{N_f} \sum_k \log\left(1 + \mathrm{e}^{-\frac{E_k}{T} - i\mathbf{q}_k\phi}\right) \right] . \tag{2.3}$$

The sum over phase space cells, k, can be, in the continuous limit, replaced by an integration over momentum space:

$$\sum_k \longrightarrow (2J_i + 1) \frac{V}{(2\pi)^3} \int \mathrm{d}^3 p \,. \tag{2.4}$$

The average multiplicity of the i^{th} hadron type can be obtained from the partition function by introducing the fictitious fugacity factor λ_i

$$\langle n_i \rangle = \frac{\partial}{\partial \lambda_i} \log \left(\mathcal{Z}(\mathbf{Q}_0,\, \lambda_i) \right) \bigg|_{\lambda_i = 1} , \tag{2.5}$$

thus

$$\langle n_i \rangle = (2J_i + 1) \frac{V}{(2\pi)^3} \frac{1}{(2\pi)^3} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \mathrm{d}^3\phi \int \mathrm{d}^3 p \left[\mathrm{e}^{E/T}\, \mathrm{e}^{i\mathbf{q}_i\phi} \pm 1 \right]^{-1} , \tag{2.6}$$

where the upper sign is for fermions and the lower is for bosons. Because the $\mathrm{e}^{-E/T}$ factor is expected to be small for all particles except pions ($T \approx 100\,\mathrm{MeV}$) the following approximation can be made in such cases

$$\frac{1}{\mathrm{e}^{E/T}\mathrm{e}^{i\mathbf{q}_i\phi} \pm 1} \longrightarrow \mathrm{e}^{-E/T - i\mathbf{q}_i\phi} \tag{2.7}$$

and then

$$\begin{aligned} \langle n_i \rangle &\approx \frac{1}{\mathcal{Z}(\mathbf{Q}_0)} \frac{1}{(2\pi)^3} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \mathrm{d}^3\phi \mathcal{Z}(\mathbf{Q}_0) \mathrm{e}^{-i\mathbf{q}_i\phi} (2J_i + 1) \frac{V}{(2\pi)^3} \int \mathrm{d}^3 p \mathrm{e}^{-E/T} \\ &= \frac{\mathcal{Z}(\mathbf{Q}_0 - \mathbf{q}_i)}{\mathcal{Z}(\mathbf{Q}_0)} (2J_i + 1) \frac{V}{(2\pi)^3} \int \mathrm{d}^3 p \mathrm{e}^{-E/T} . \end{aligned} \tag{2.8}$$

Very good agreement with the measured particle ratios was found in, e.g., [14]. Eqs. (2.6), (2.8) can also be used to calculate the respective transverse momentum distributions for pions and heavier hadrons produced in the thermodynamical hadronization process.

The high $p_\perp$ tails of these distributions are known [3] to fall like

$$f(p_\perp) \xrightarrow[p_\perp \gg m, T]{} {p_\perp}^{3/2} \mathrm{e}^{-p_\perp/T} , \tag{2.9}$$

what is in agreement with the low energy experimental results but there is a clear underestimate of the high $p_\perp$'s at collider energies.

3. Modifications of the statistical hadronization model

The statistical way is "phenomenological" in a sense. The conventional Boltzmann-Gibbs description shown above should be modified, and a new parameter describing the correlation "strength", however defined, ought to be introduced. Of course, in the limit of the absence of correlations the new description should approach the Boltzmann form.

From the theoretical point of view there could be infinitely many "generalized" statistics. Following 'Ockham razor' we should choose that which is simple and has a clear theoretical background. In the present paper we test the possibility, proposed by Tsallis [15], based on the modification of the classical entropy definition

$$S_{\mathrm{BG}} = -k \sum_i^W p_i \ln p_i \tag{3.1}$$

of the form

$$S_q = k\frac{1-\sum_i^W p_i^q}{q-1} \tag{3.2}$$

introducing the new parameter q, the non-extensivity parameter. This modification has been adopted in other physical applications (see, e.g., [16]).

Maximization of the entropy requirement with the total energy constraint

$$\frac{\sum_i p_i^q E_i}{\sum_i p_i^q} = E_0 \tag{3.3}$$

leads to the probability given by

$$p_i^q = \frac{1}{Z_q}\left[1 - \frac{(1-q)}{T_q(E_i-E_0)}\right]^{q/(1-q)}, \tag{3.4}$$

where Z_q is the normalization constant related to $\mathcal{Z}(q)$ of eq. (2.1) and the Boltzmann terms are replaced by the probabilities of the form given in eq. (3.4).

It can be mentioned here, that the name of this generalization: "the non-extensive statistics" comes from the fact that the entropy S_q defined by eq. (3.2), in opposition to the S_{BG}, is non-extensive parameter (the entropy of the system consisting of two separated parts is not a sum of their entropies). A similar statement is valid for the total energy of the system. If it consists of n isolated parts (e.g., gas of "non-interacting" hadrons), each of energy E_i, the system total energy is not equal to $\sum_i^n E_i$ but given by

$$E = \sum_i^n E_i + \frac{(q-1)}{T\sum_{i,j} E_i E_j} + \frac{(q-1)^2}{T^2\sum_{i,j,k} E_i E_j E_k} + \cdots . \tag{3.5}$$

Additional terms can be interpreted as an effect of the long-range correlation which appears to be not only the mathematical construction adopted here, but has something to do with the real cause of the correlation.

Equation (3.4) can be rewritten introducing new symbol, e_q, defined as

$$(\mathrm{e}_q)^x = [1+(1-q)x]^{q/(1-q)} \tag{3.6}$$

(for completeness, in the $q=1$ limit we have, as we should, $\mathrm{e}_1^x = \mathrm{e}^x$). Then

$$p_i^q \sim \left[1 - \frac{(1-q)}{T_q(E_i-E_0)}\right]^{q/(1-q)} \sim \mathrm{e}_q^{-E_i/T_q} \tag{3.7}$$

and the partition function can be written in the form

$$\mathcal{Z}_q(\mathbf{Q}) = \sum_{\text{states}} \frac{1}{(2\pi)^3}\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi} \mathrm{d}^3\phi\, \mathrm{e}_q^{-E/T} \mathrm{e}^{i(\mathbf{Q}_0-\mathbf{Q})\phi} . \tag{3.8}$$

With such a modification, the transverse momentum distribution becomes

$$f(p_\perp) \sim \frac{\mathcal{Z}_q(\mathbf{Q}_0-\mathbf{q}_i)}{\mathcal{Z}_q(\mathbf{Q}_0)}(2J_i+1)\frac{V}{(2\pi)^3}\int \mathrm{d}p_{||}p_\perp \mathrm{e}_q^{-E/T} \tag{3.9}$$

JHEP12(2003)039

for particles other than pions, while for pions the modified eq. (2.6) should be used:

$$f(p_\perp) \sim (2J_i+1)\frac{V}{(2\pi)^3}\frac{1}{(2\pi)^3}\int_0^{2\pi}\int_0^{2\pi}\int_0^{2\pi} \mathrm{d}^3\phi \int \mathrm{d}p_{||}p_\perp \left[\mathrm{e}_q^{E/T}\mathrm{e}^{i\mathbf{q}_i\phi}-1\right]^{-1}, \qquad (3.10)$$

(the vector $\mathbf{q}_i$ in the above equations represents the i^{th} particle type and q is the non-extensivity parameter).

In the present work we have evaluated $\mathcal{Z}_q$ functions for a variety of its parameters T, V, and q, and for $\mathbf{Q}$ values which cover the production of over 100 hadrons of masses below 2 GeV. The decays of short-lived particles were then performed (with three-body decay products distributed uniformly in the Dalitz plot). The effect on the $p_\perp$ distribution caused by the decays is shown in figure 2

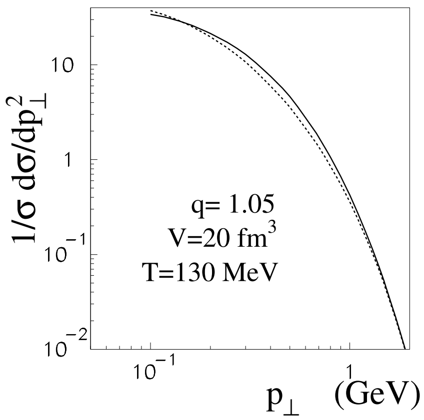

Figure 2: Distributions of $p_\perp$ for charged primary produced hadrons (dashed line) and for long-lived hadrons after all decays (solid line) ($T = 130\,\mathrm{MeV}$, $V = 20\,\mathrm{fm}^3$ and $q = 1.05$). (Strictly, the abscissa, $p_\perp$, should be in units of GeV/c but we disregard the 'c' here and elsewhere.)

Examples of final transverse momentum distributions are shown in figure 3 for different thermodynamical parameters: T, V, and q. Closer inspection of the $p_\perp$ distributions gives reasons to reduce the number of (independent) parameters which should be used when comparing model calculations with the measured data in the high $p_\perp$ region. The dependence on the hadronization volume V for not very small $p_\perp$, as seen in the left graph in figure 3, is not crucial. Additionally, the parameter T (hereafter it will be called 'temperature', bearing in mind that in the non-extensive thermodynamics, for $q > 1$, its meaning is not so obvious) and V are strongly correlated. In ref. [14], for the fitting procedure, the parameter VT^3 has been chosen instead of V. The main subject of the present paper is not the multiplicity, but rather the shape of the transverse momentum distribution (and mainly for high transverse momenta), thus the normalization in the $p_\perp$ region of interest is quite natural to be used. The temperature influences mainly relatively small momenta, not much higher than its actual value, if, of course, the normalization is treated separately. The general normalization factor combines both T and V dependencies and, as detailed calculations confirm, when analyzing high $p_\perp$ data (about and above $\sim$1 GeV/c), the parameter V can be successfully replaced by the overall normalization parameter with only a slight change in two remaining parameters: T and q.

The non-extensivity parameter q, the crucial one for the present work, determines the asymptotic index of the high $p_\perp$ distribution tails. The following approximate formula found in [16] illustrates this

$$f(p_\perp) \sim p_\perp \int_0^\infty \mathrm{d}p_{||}\left[1+\frac{(1-q)}{T\sqrt{{p_{||}}^2+{p_\perp}^2+m^2}}\right]^{-q/(q-1)}$$

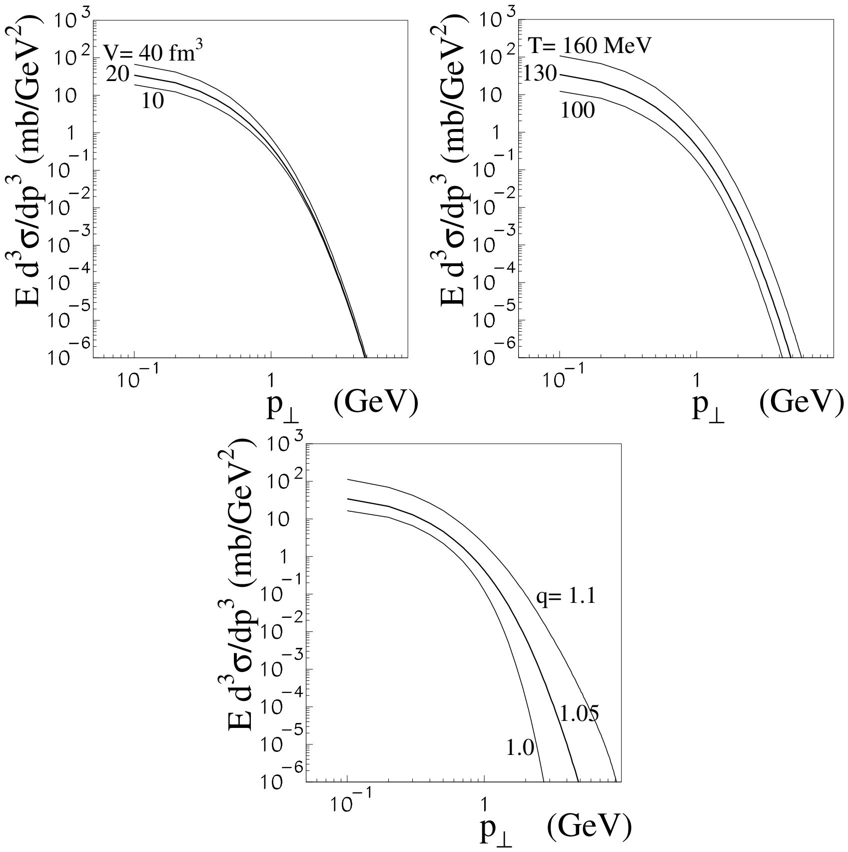

Figure 3: Distributions of $p_\perp$ for charged long-lived hadrons calculated for different V, T, and q values around $T = 130\,\mathrm{MeV}$, $V = 20\,\mathrm{fm}^3$ and $q = 1.05$.

$$\sim \left(\frac{p_\perp}{T}\right)^{3/2} \left[1 + \frac{p_\perp(q-1)}{T}\right]^{-\frac{q}{q-1}+\frac{1}{2}} . \tag{3.11}$$

We will compare it with our exact calculation results.

Investigation of the energy dependence of q should give us the answer to the question as to whether the modified thermodynamical model can be reasonably applied as a phenomenological description of complex QCD principles "at work" in the hadronization process.

4. Application to the data

The non-extensive framework was successfully applied to transverse momentum data for $e^+e^- \to$ hadrons in [17]. In the present work we present results concerning pp (and $p\bar{p}$) reactions. The available data covered quite a wide range of particle interaction energies. We have started our analysis at $p_{\mathrm{lab}} = 100\,\mathrm{GeV/c}$ [18] where the $p_\perp$ distributions match quite nicely the exponential behaviour and then through [19] and ISR energies [20, 21], SPS [22] and [23] up to two Tevatron energies: $\sqrt{s} = 630$ and $1800\,\mathrm{GeV}$ [24, 25].

Our model parameters (temperature T and non-extensivity q) have been adjusted (for $p_\perp > 0.5\,\mathrm{GeV}$) to describe measured invariant $p_\perp$ distributions. The absence of any systematic change of temperature was found. Thus we set the value of T equal to 130 MeV and performed the minimization procedures again.

The first important point we have to mention here is that the reproduction of the data is very good.

The next finding is a systematic increase of the non-extensivity parameter starting from the value of 1 (i.e., exponential, e.g., classical, $p_\perp$ distribution tails) at $\sqrt{s} \approx 10$ GeV.

The near-perfect match as obtained suggests a check to see if there exists any evidence at all, that the non-extensivity parameter may have a non-unique value for a fixed interaction energy. Some interesting statements, suggesting such q behaviour (but for the longitudinal phase space) have been published recently [26]. For example, it is possible that the non-extensivity varies with the actual multiplicity of created particles, if so it could be related somehow to, for example, the impact parameter of the colliding hadrons [27].

The spread of q was allowed to be gaussian (with the mean value and dispersion as free parameters to be adjusted). Higher degree polynomials have also been tested, but no improvement was found.

The fitting procedure was repeated again for all data sets listed above with the T parameter released free again at the first step. No important energy dependence was noticed here either, so we fixed it again at 130 MeV.

The final transverse momentum distributions are presented in figure 4

The distributions of the q parameter starts from $\delta(q-1)$ for sub-ISR energies, and then, when the average q differs from 1 they become gaussian (truncated below 1). Some examples are presented in figure 5. In all cases the widths of the q distributions are very small (their dispersions are about 0.01). This allows us to claim that the transverse momentum distributions for a given interaction energy can be well described by a single value for the non-extensivity parameter. This means that the parameter does not depend on the actual multiplicity (impact parameter) or any other interaction characteristic which may fluctuate from event to event. Its value is determined only by the available center of mass energy. It should be mentioned here, that this statement supports the non-extensive thermodynamical treatment of the hadronization process making it simple and clear.

However, most important is the intriguing regularity as seen in the figure 6 where we show how the non-extensivity parameter q depends on the interaction energy.

The value of q should not exceed 1.25 [10], and it is quite reasonable to expect this as an asymptotic limit leading to QCD-inspired $p_\perp^{-4}$ distribution for large transverse momenta at extremely high energies. The simple dependence is found to be of the form

$$q = 1.25 - 0.33 s^{-0.054} \tag{4.1}$$

as shown in figure 6.

5. Rise of the average transverse momenta

The average transverse momentum of particles created in high energy hadronic interactions in the non-extensive thermodynamical model increases when the non-extensivity parameter

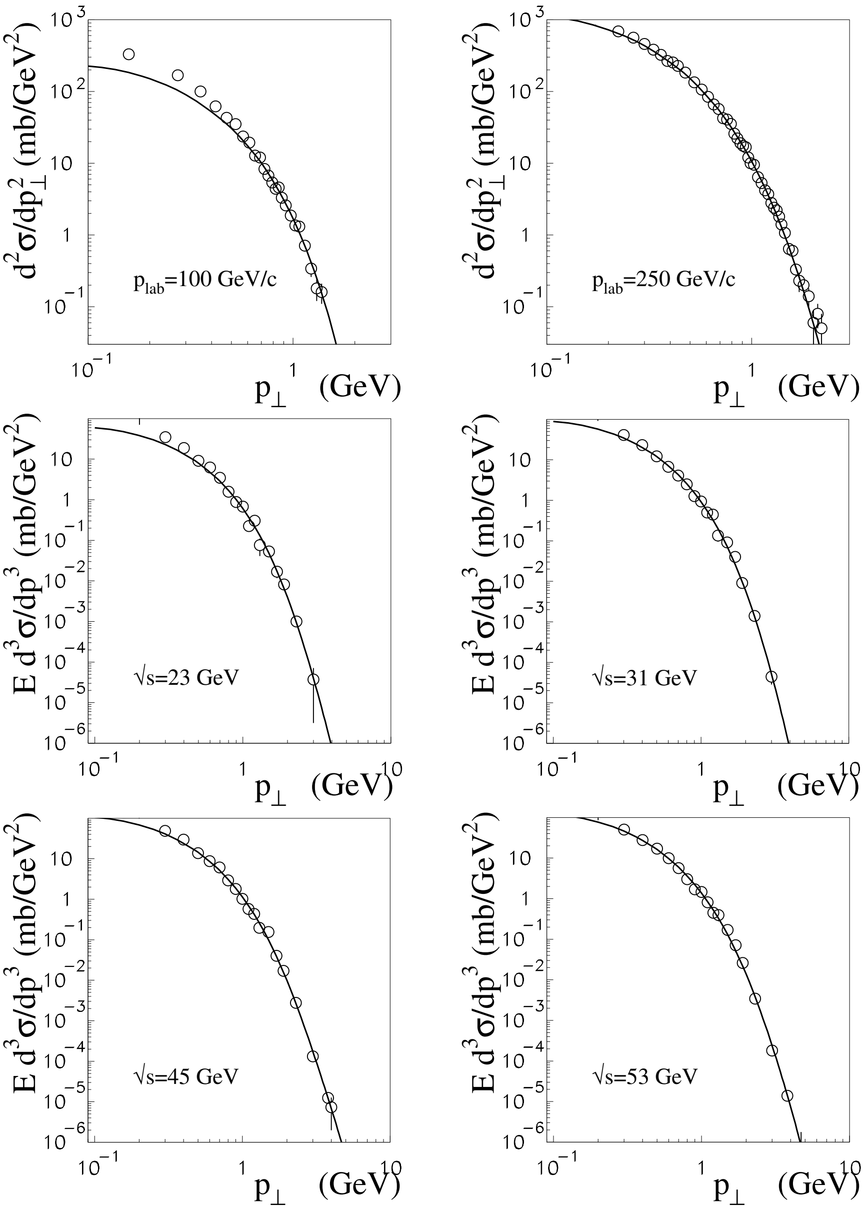

Figure 4: Transverse momentum distributions measured at different energies compared with non-extensive fits.

increases. In [16] the average of the distribution given by eq. (3.11) was calculated. The form of the dependence is

$$\langle p_\perp \rangle = T \frac{1}{2} \frac{5}{4-3q} \,. \tag{5.1}$$

This is, however, an approximate result (as is eq. (3.11)). The approximation is quite good but it can only give the average value for primary created hadrons. We have obtained the average $p_\perp$ exactly and corrected for all the effects of decays of short-lived resonances. It is presented in figure 7 where eq. (5.1) is compared with the results obtained from the fits presented in figure 4.

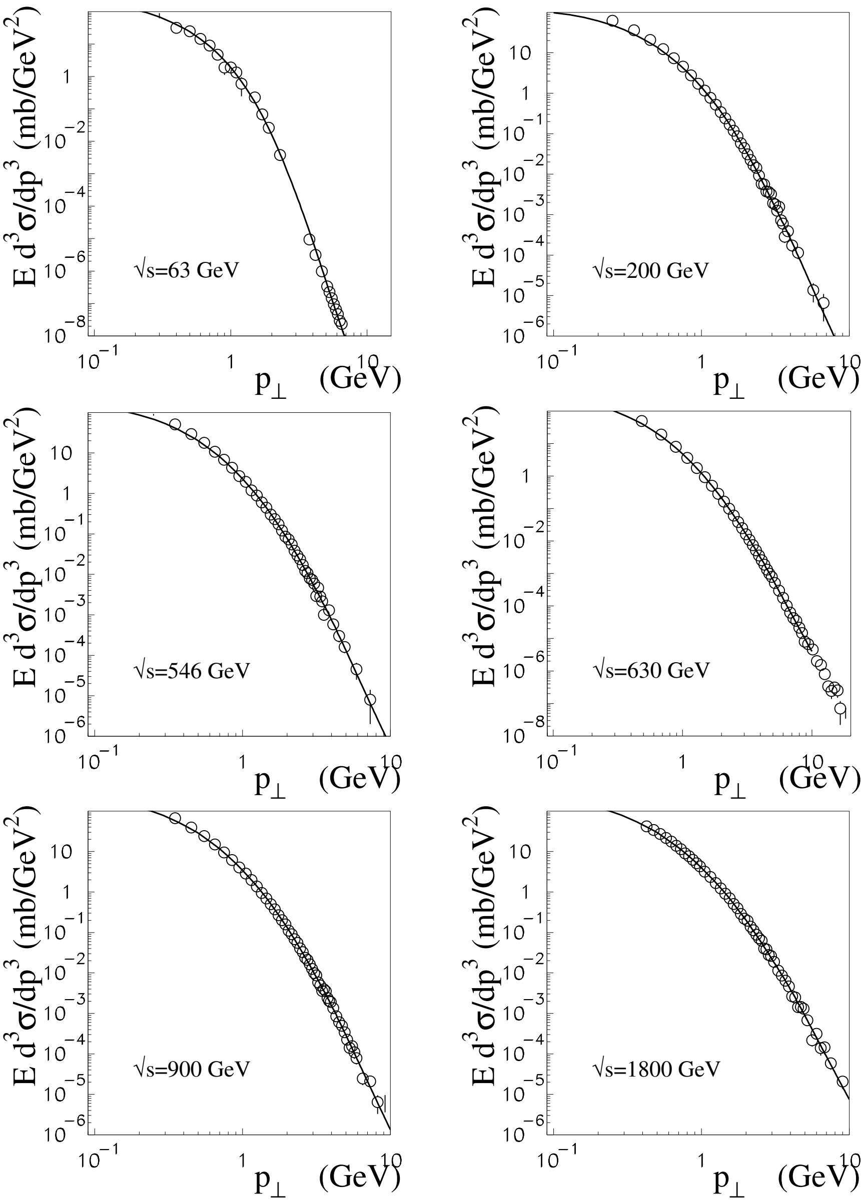

Figure 4: (cont.) Transverse momentum distributions measured at different energies compared with non-extensive fits.

Using eq. (4.1) we can present the average $p_\perp$ as a function of energy; the result is shown in figure 8.

The existence of the 1.25 asymptotic limit for q and the clear and well defined rise of q observed in the data allows us to extrapolate the results to much higher energies with a high degree of confidence. The line plotted in figure 8 shows a moderate $\langle p_\perp \rangle$ increase (as $s^{0.037}$ or of about 0.1 GeV per decade of the center of mass available energy) for large interaction energies.

This is relevant for the creation of new Monte Carlo generators and interaction models used in cosmic ray physics simulations of extensive air showers (EAS) at very high energies.

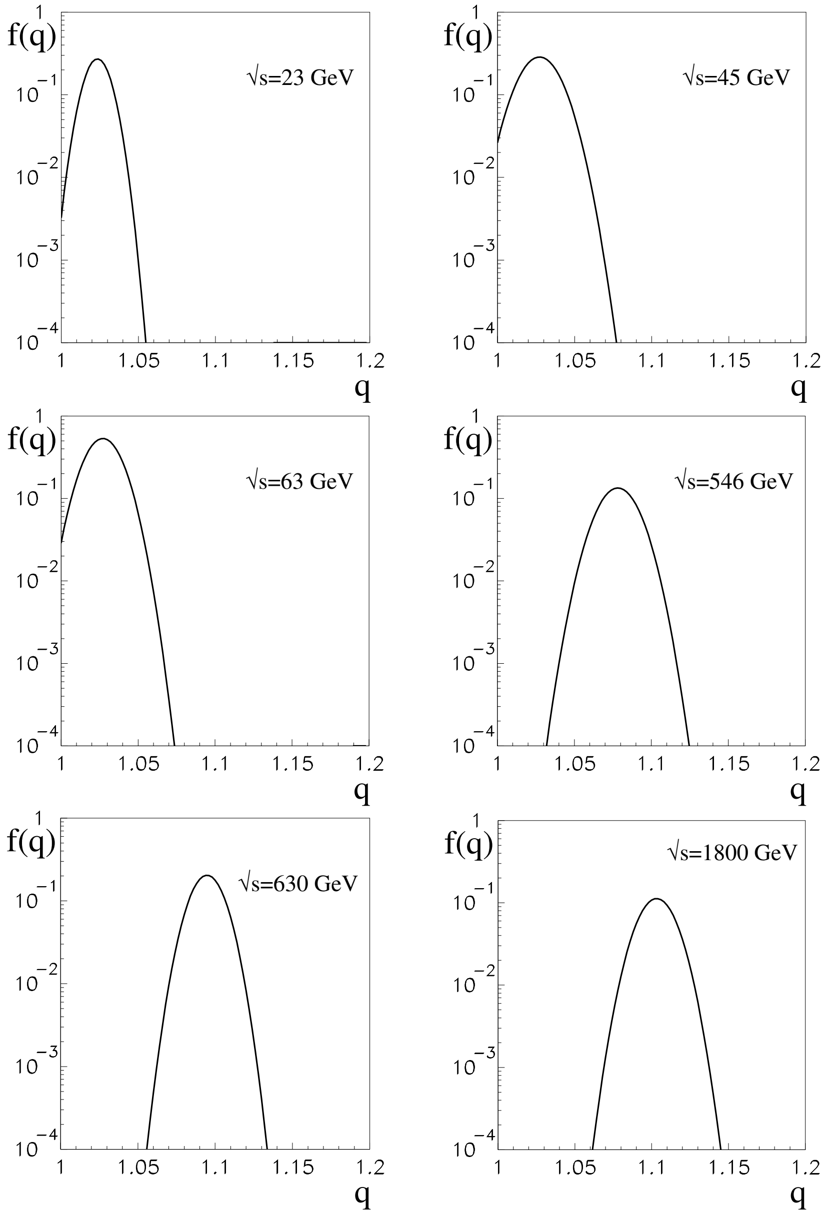

Figure 5: Some examples of distributions of the non-extensivity parameter q giving the best description of the data shown in figure 4.

Just now is the moment when the new generation of giant EAS arrays are built and new data from, e.g., Auger Observatory for energies as high as 10^{20} eV are expected [28]. Their proper interpretation and the accurate primary particle energy estimation requires detailed knowledge of the expected lateral particle distribution which is determined in part by the transverse particle spread high in the atmosphere where the particles interact. The energies are, of course, much above those energy created at the accelerators.

According to [29] the average $p_\perp$ used in all models implemented in the CORSIKA code, one of the most widely used EAS simulation programs, at 10^{19}eV is 0.55 GeV while our fit suggests a value which is about 20% higher.

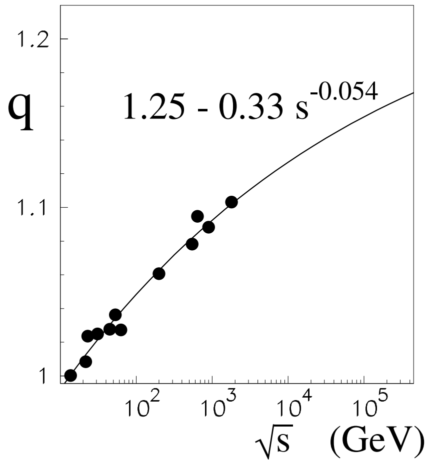

Figure 6: Energy dependence of the average value of the non-extensivity parameter q obtained from the $p_\perp$ distributions. The functional fit is shown by the line. Uncertainties at each point coming from the fitting procedure are much smaller than plotted points. The spread with respect to the line is a result of differing systematic uncertainties in the $p_\perp$ distributions measured by different experiments.

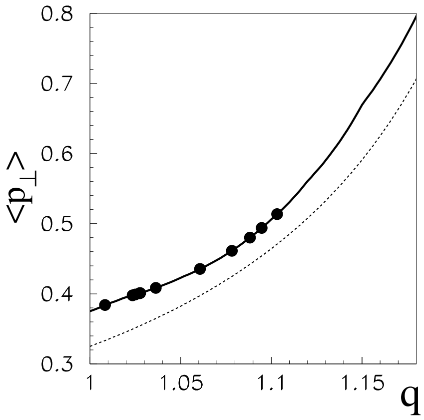

Figure 7: The calculated average transverse momentum of long-lived charged hadrons as a function of the non-extensivity parameter q. The dashed line shows the approximation given by eq. (5.1). The points represent the mean $p_\perp$'s calculated from the fits shown in figure 4.

Cosmic ray experiments have been delivering for a very long time data related to average transverse momenta at energies exceeding the contemporary accelerator abilities. The most straightforward, at first sight, are the calorimeter experiments at mountain altitudes. The claim of an abrupt and very substantial $p_\perp$ increase at around $\sqrt{s} = 1000\,\mathrm{GeV}$ made in [30] was based on observations of events with large values of the Er product in the Tien-Shan hadronic calorimeter. The registration of the shower of particles and determination of their energies E and distance to the shower axis r enables a determination of the distribution of product Er which is related to the transverse momentum by

$$p_\perp = \frac{r}{h}\frac{E}{c}\,, \tag{5.2}$$

where h is the actual hadron production height. However, methodological difficulties in the data interpretation (see, e.g, [31]) do not permit such a strong statement. In spite of this fact Er is still the simplest and promising way to study the transverse spread of hadrons created in very high energy collisions.

In [32] the distribution of Er from the joint Chacaltaya-Pamir experiment was published, again with the conclusion that its shape could not be fully explained by the simulations. Their data are shown in figure 9 as solid circles and the results of our calculations with two interaction models are given by the respective histograms. For the shower simulation we used the structure of the CORSIKA program [33]. The well known FRITIOF [34] model was implemented to see if the event generator widely used in accelerator physics

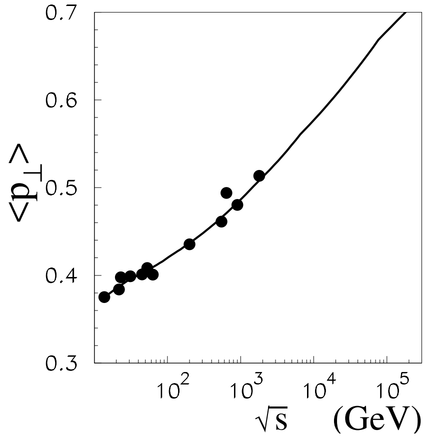

Figure 8: The average transverse momentum of long-lived charged hadrons as a function of the available interaction center of mass energy for our model with $q(s)$ given by eq. (4.1). The points are as in figure 7.

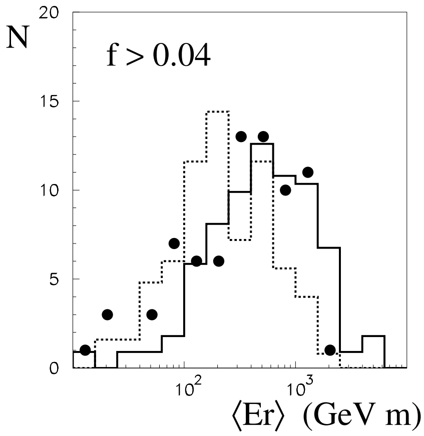

Figure 9: Distribution of $\langle Er \rangle$ in the Chacaltaya-Pamir experiment [32] compared with predictions of FRITIOF (dashed histogram) and modified HDPM (solid one) models.

works well also in the cosmic ray domain. The second model was the version of the default CORSIKA model called HDPM ([35] with further improvements) in which we change the generation of transverse momenta according to the results obtained in the present paper.

It is seen that the data, at least at high $\langle Er \rangle$, can be explained by the simulations. The FRITIOF generator does not look very well here, but it do not have to be only the problem of transverse momenta. For the emulsion chamber data the very forward region of particle creation is essential and the FRITIOF was rather tuned (and the ARIADNE [36] which is the part of it responsible for hard gluon bremsstrahlung) for other sorts of data. This is seen in figure 10 where we plot the distribution of the energy fraction carried by the energetic photons (γ quanta) in the γ-hadron families observed by the Chacaltaya experiment.

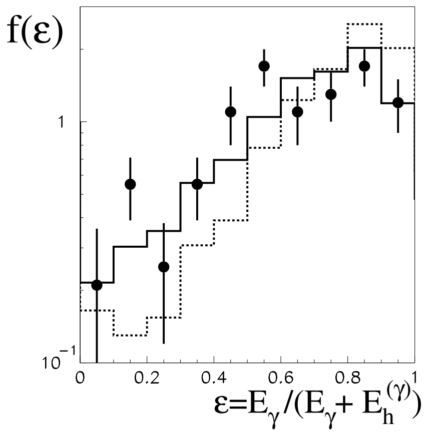

Figure 10: Distribution of electromagnetic energy fraction seen in hadron families at mountain altitude cosmic ray Chacaltaya experiment [37]. The dashed histogram represents FRITIOF model prediction while the solid one is the modified version of HDPM algorithm.

6. Conclusions

We have applied modified, non-extensive statistical thermodynamics to the data on particles with high transverse momenta created in high energy hadronic collisions. It has been found that the data can be successfully described with a fixed temperature parameter T and a non-extensivity parameter q rising slowly with interaction energy.

The non-extensive thermodynamics can be used to describe phenomenologically the long-range correlations (e.g., jet phenomena) without introducing particular physics. The achieved agreement for one q value depending only on the interaction energy confirms such interpretation of the non-extensivity.

The rise of q leads to systematic rise of the average transverse momentum. The regularity

$$q = 1.25 - 0.33\, s^{-0.054}$$

allows us to predict with some confidence the rise of $\langle p_\perp \rangle$ and extrapolate it to the energies exceeding these available at present accelerators up to the highest cosmic ray energies. The importance of the exactness of such an extrapolation is clear in view of the fact that extensive data from the Auger Observatory are expected soon. The interpretation and understanding of EAS features at primary particle energies exceeding 10^{20}eV is a clue to the solution of one of the most exciting astrophysical problems.

References

[1] A. Bialas, *Fluctuations of string tension and transverse mass distribution*, *Phys. Lett.* **B 466** (1999) 301 [hep-ph/9909417].

[2] E. Fermi, *High-energy nuclear events*, *Prog. Theor. Phys.* **5** (1950) 570.

[3] R. Hagedorn, *Remarks on the thermodynamical model of strong interactions*, *Nucl. Phys.* **B 24** (1970) 93.

[4] R. Hagedorn and J. Ranft, *Inclusive single particle spectra and the strong thermodynamic bootstrap solution*, *Nucl. Phys.* **B 48** (1972) 157.

[5] R.P. Feynman, R.D. Field and G.C. Fox, *Correlations among particles and jets produced with large transverse momenta*, *Nucl. Phys.* **B 128** (1977) 1.

[6] R.D. Field and R.P. Feynman, *A parametrization of the properties of quark jets*, *Nucl. Phys.* **B 136** (1978) 1.

[7] R.D. Field and R.P. Feynman, *Quark elastic scattering as a source of high transverse momentum mesons*, *Phys. Rev.* **D 15** (1977) 2590.

[8] R.P. Feynman, R.D. Field and G.C. Fox, *Quantum-chromodynamic approach for the large-transverse-momentum production of particles and jets*, *Phys. Rev.* **D 18** (1978) 3320.

[9] F. Becattini, L. Bellucci and G. Passaleva, *Transverse momentum spectra of identified particles in high energy collisions with statistical hadronisation model*, *Nucl. Phys.* **92** *(Proc. Suppl.)* (2001) 137 [hep-ph/0010221].

[10] C. Beck, *Nonextensive methods in turbulence and particle physics*, *Physica* **A305** (2002) 209 [cond-mat/0110071].

[11] G. Wilk and Z. Wlodarczyk, *Application of nonextensive statistics to particle and nuclear physics*, *Physica* **A305** (2002) 227 [hep-ph/0108215].

[12] G. Wilk and Z. Wlodarczyk, *Traces of nonextensivity in particle physics due to fluctuations*, hep-ph/0210175.

[13] R. Hagedorn, *Hadronic matter near the boiling point*, *Nuovo Cim.* **A56** (1968) 1027.

JHEP12(2003)039

[14] F. Becattini and U.W. Heinz, *Thermal hadron production in PP and $p\bar{p}$ collisions*, *Z. Physik* **C 76** (1997) 269 [hep-ph/9702274].

[15] C. Tsallis, *Possible generalization of Boltzmann-Gibbs statistics*, *J. Stat. Phys.* **52** (1988) 479.

[16] C. Beck, *Non-extensive statistical mechanics and particle spectra in elementary interactions*, *Physica* **A286** (2000) 164 [hep-ph/0004225].

[17] I. Bediaga, E.M.F. Curado and J.M. de Miranda, *A nonextensive thermodynamical equilibrium approach in $e^+e^- \rightarrow$ hadrons*, *Physica* **A286** (2000) 156 [hep-ph/9905255].

[18] C.P. Ward et al., *General features of charged particle production in $\bar{p}p$ interactions at 100 GeV/c*, *Nucl. Phys.* **B 153** (1979) 299.

[19] EHS-NA22 collaboration, M. Adamus et al., *Charged particle production in K^+p, π^+p and pp interactions at 250 GeV/c*, *Z. Physik* **C 39** (1988) 311.

[20] British-Scandinavian collaboration, B. Alper et al., *Production spectra of $\pi^\pm$, $K^\pm$, $\rho^\pm$ at large angles in proton proton collisions in the cern intersecting storage rings*, *Nucl. Phys.* **B 100** (1975) 237.

[21] CERN-Dortmund-Heidelberg-Warsaw collaboration, D. Drijard et al., *A measurement of the inclusive cross-section of charged pions at very high transverse momenta*, *Nucl. Phys.* **B 208** (1982) 1.

[22] UA1 collaboration, C. Albajar et al., *A study of the general characteristics of proton–anti-proton collisions at $\sqrt{s} = 0.2$ TeV to 0.9 TeV*, *Nucl. Phys.* **B 335** (1990) 261.

[23] UA2 collaboration, M. Banner et al., *Inclusive particle production in the transverse momentum range between 0.25 GeV/c and 40 GeV/c at the Cern $sp\bar{p}s$ collider*, *Z. Physik* **C 27** (1985) 329.

[24] G. Bocquet et al., *Inclusive production of strange particles $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV with UA1*, *Phys. Lett.* **B 366** (1996) 441.

[25] CDF collaboration, F. Abe et al., *Transverse momentum distributions of charged particles produced in $\bar{p}p$ interactions at $\sqrt{s} = 630$ GeV and 1800 GeV*, *Phys. Rev. Lett.* **61** (1988) 1819.

[26] F.S. Navarra, O.V. Utyuzh, G. Wilk and Z. Wlodarczyk, *Estimating the inelasticity with the information theory approach*, *Phys. Rev.* **D 67** (2003) 114002 [hep-ph/0301258].

[27] T. Wibig, *High-energy interactions around the 'knee'*, *J. Phys.* **G 27** (2001) 1633.

[28] Pierre Auger collaboration, J. Blumer, *Cosmic rays at the highest energies and the Pierre Auger observatory*, *J. Phys.* **G 29** (2003) 867.

[29] J. Knapp, D. Heck and G. Schatz, *Comparison of hadronic interaction models used in air shower simulations and of their influence on shower development and observables*, FZKA-5828, Karlsruhe 1996.

[30] M.M. Winn, *Giant air showers and EAS structure*, in *15th Internat. Cosmic Ray Conf. Plovdiv* **10** (1977) 305.

[31] J.N. Capdevielle et al., *High energy hadrons in eas at mountain altitude*, *J. Phys.* **G 24** (1998) 1785 [hep-ph/9808239].

[32] Chacaltaya-Pamir collaboration, L.T. Baradzei et al., *A study of very high energy hadron interaction through observation of cosmic-ray families of visible energy greater than 500 TeV [1]*, in *23rd Internat. Cosmic Ray Conf.*, Calgary **4** (1993) 116.

[33] D. Heck, G. Schatz, T. Thouw, J. Knapp and J.N. Capdevielle *CORSIKA: a Monte Carlo Code to simulate extensive air showers*, FZKA-6019, Karlsruhe 1998.

[34] B. Andersson, G. Gustafson and H. Pi, *The FRITIOF model for very high-energy hadronic collisions*, *Z. Physik* **C 57** (1993) 485.

[35] J.N. Capdevielle, *A Monte Carlo generator for cosmic ray collisions*, *J. Phys.* **G 15** (1989) 909.

[36] L. Lonnblad, *The colour-dipole model and the ariadne program at high* Q^2, `hep-ph/9908368`.

[37] C.E. Navia, C.R.A. Augusto, F.A. Pinto and E.H. Shibuya, *Hadron-rich cosmic ray families detected by emulsion chamber*, *Nuovo Cim.* **A108** (1995) 1341.